In case there are $n$ inputs, that is, if each *if*-side contains $n$ variables, the relation matrix **R** generalises to an $n + 1$ dimensional array. Let $\mathbf{e}_i$ $(i = 1, \ldots, n)$ be the inputs, then inference is carried out by a generalised composition,

$$\mathbf{u} = (\mathbf{e}_1 \times \mathbf{e}_2 \times \ldots \times \mathbf{e}_n) \vee . \wedge \mathbf{R}$$

Inference is still the usual composition operation; we just have to keep track of the dimensions.

## 5. Summary

We have achieved a method of representing and executing a rule $If\ the\ level\ is\ low\ then\ open\ V_1$ in a computer program. In summary:

1. Define fuzzy sets **low** and **open** corresponding to a low level and an open valve; these can be defined on different universes.
2. Represent the implication as a relation **R** by means of the outer product, $\mathbf{R} =$ **low** $\circ$.**min** **open**. The result is a matrix.
3. Perform the inference with an actual measurement. In the most general case this measurement is a fuzzy set, say, the vector **level**. The control action $\mathbf{v}_1$ is obtained by means of the compositional rule of inference, $\mathbf{v}_1 =$ **level** $\vee . \wedge$ **R**.

Fuzzy controllers are implemented in a more specialised way, but they were originally developed from the concepts and definitions presented above, especially inference and implication.

## References

Franksen, O. I. (1978). Group representation of finite polyvalent logic, *in* A. Niemi (ed.), *Proceedings 7th Triennial World Congress*, International federation of automatic control, IFAC, Pergamon, Helsinki.

Fukami, S., Mizumoto, M. and Tanaka, K. (1980). Some considerations of fuzzy conditional inference, *Fuzzy Sets and Systems* **4**: 243–273.

Holmblad, L. P. and Østergaard, J.-J. (1982). Control of a cement kiln by fuzzy logic, *in* Gupta and Sanchez (eds), *Fuzzy Information and Decision Processes*, North-Holland, Amsterdam, pp. 389–399. (Reprint in: FLS Review No 67, FLS Automation A/S, Høffdingsvej 77, DK-2500 Valby, Copenhagen, Denmark).

Jantzen, J. (1995). Array approach to fuzzy logic, *Fuzzy Sets and Systems* **70**: 359–370.

Kaufmann, A. (1975). *Introduction to the theory of fuzzy sets*, Academic Press, New York.

Kiszka, J. B., Kochanska, M. E. and Sliwinska, D. S. (1985). The influence of some fuzzy implication operators on the accuracy of a fuzzy model, *Fuzzy Sets and Systems* **15**: (Part1) 111–128; (Part 2) 223 – 240.

Lee, C. C. (1990). Fuzzy logic in control systems: Fuzzy logic controller, *IEEE Trans. Systems, Man & Cybernetics* **20**(2): 404–435.

Mamdani, E. H. (1977). Application of fuzzy logic to approximate reasoning using linguistic synthesis, *IEEE Transactions on Computers* **C-26**(12): 1182–1191.

Mizumoto, M., Fukami, S. and Tanaka, K. (1979). Some methods of fuzzy reasoning, *in* Gupta, Ragade and Yager (eds), *Advances in Fuzzy Set Theory Applications*, North-Holland, New York.

Singh, M. G. (ed.) (1987). *Systems and Control Encyclopedia: Theory, Technology, Applications*, Pergamon, Oxford.

Singh, M. G. (ed.) (1990). *Systems and Control Encyclopedia: Theory, Technology, Applications*, Vol. Supplementary volume 1, Pergamon, Oxford.

Singh, M. G. (ed.) (1992). *Systems and Control Encyclopedia: Theory, Technology, Applications*, Vol. Supplementary volume 2, Pergamon, Oxford.

Wenstøp, F. (1980). Quantitative analysis with linguistic values, *Fuzzy Sets and Systems* **4**(2): 99–115.

Yager, R. R., Ovchinnikov, R., Tong, M. and Nguyen, H. T. (eds) (1987). *Fuzzy sets and applications - selected papers by L. A. Zadeh*, Wiley and Sons, New York etc.

Zadeh, L. A. (1965). Fuzzy sets, *Inf. and control* **8**: 338–353.

Zadeh, L. A. (1973). Outline of a new approach to the analysis of complex systems and decision processes, *IEEE Trans. Systems, Man & Cybernetics* **1**: 28–44.

Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning, *Information Sciences* **8**: 43–80.

Zadeh, L. A. (1984). Making computers think like people, *IEEE Spectrum* pp. 26–32.

Zadeh, L. A. (1988). Fuzzy logic, *IEEE Computer* **21**(4): 83–93.

Zimmermann, H.-J. (1993). *Fuzzy set theory - and its applications*, second edn, Kluwer, Boston. (1. ed. 1991).